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Abstract

This paper analyses the methodology developed by Behrens and Murata (2007) to introduce variable mark-ups into models of monopolistic competition. Their risk-aversion explanation to the presence of fixed mark-ups in the Dixit and Stiglitz (1977) model is validated; however, we show that their constant absolute risk aversion solution ignores existing mechanisms found in the new Keynesian literature. From these we develop a model of new economic geography with a variable elasticity of substitution and variable mark-ups consistent with Behrens and Murata (2007). However, we argue that from both a theoretical and empirical perspective this new Keynesian approach is preferable to the solution of Behrens and Murata (2007).

JEL classification: D43; F12; L13; R12; R13.

Keywords: New economic geography; Variable mark-ups; Monopolistic competition; Flexible varieties aggregator.

1 Introduction

A well-known property of the standard constant elasticity of substitution (CES) model of monopolistic competition developed by Dixit and Stiglitz (1977) is that it displays constant mark-ups regardless of the number of firms competing. Within new economic geography (NEG), where the Dixit-Stiglitz approach is very popular, this is remarked upon by Krugman (1998):

> "The assumed symmetry amongst varieties and the resulting absence [...] of any strategic behaviour by firms means that Dixit-Stiglitz undoubtedly misses much of what really happens in imperfectly competitive industries"

The general intuition behind such models of monopolistic competition is that in the presence of a preference for variety, product differentiation offers the possibility for firms to protect their market power from competitors. However, a mark-up that is independent from an increase in the number of competitors implies that the protection offered by increased product differentiation perfectly offsets the increased competition. As emphasised by Behrens and Murata (2007), this is a very strong assumption, and one that is unlikely to be observed in practice. This problem is therefore of particular importance in NEG. Typically, mark-ups and pricing policies are predicted to be independent both of the number of producers and their location. Empirically, however, in the presence of agglomeration one could expect mark-ups to exhibit some sort of spatial structure, or equivalently one could expect to see pricing-to-market behaviour from firms.

Behrens and Murata (2007) address this issue by developing a model of monopolistic competition which incorporates pro-competitive effects, including a competitive limit whereby marginal cost pricing occurs when the firm mass tends to infinity. It is important to emphasise from the start that the arguments they raise concerning the limits of the CES case are valid, and the constant absolute risk aversion (CARA) approach they propose provides a tractable way of getting around this problem. Using this approach, they evaluate in Behrens and Murata (2006) the welfare aspects of the CARA approach by comparing the autarky and free trade equilibria of the model.

While the premise of Behrens and Murata (2007) is correct, the solution suggested generally ignores the existing monopolistic competition literature that is based on quasi-linear utility functions, typically Ottaviano et al. (2002) and more recently Melitz and Ottaviano (2008), which additionally integrates heterogeneous firms. More importantly from the point of view of NEG, Behrens and Murata (2007) also ignores the flexible varieties aggregator developed in Kimball (1995), which builds a simple and elegant extension to Dixit and Stiglitz (1977) that exhibits a variable elasticity of substitution (VES). Dotsey and King (2005) and Sbordone (2007) in particular provide a good illustration of how this framework can be used to incorporate pro-competitive effects in the CES framework of Calvo (1983).[1]

This paper shows that in a similar manner, the Kimball (1995) flexible varieties aggregator can be used to generate pro-competitive effects and variable mark-ups in a simple extension of standard CES NEG models. Furthermore, it will be argued that the resulting modeling approach is better suited for NEG than the Behrens and Murata (2007) approach, and possibly the quasi-linear utility models mentioned above. This is because it provides pro-competitive equilibrium and welfare predictions that are in line with Behrens and Murata (2006) while still nesting the benchmark Dixit and Stiglitz (1977) NEG model.

The remainder of the paper is organised as follows: Section 2 introduces the Kimball (1995) aggregator, in particular the use of an implicit definition of the manufacturing aggregate and adapts it to the typical NEG specification. This section shows, in agreement with Behrens and Murata (2007), the link between the elasticity of demand, mark-ups and risk aversion in such a model. The complete derivation of the VES NEG model as well as the welfare aspects of autarky and free trade are presented in section 3. Finally, section 4 concludes.

[1]Sbordone (2007) probably provides the best mathematical exposition of the Kimball (1995) framework, as well as illustrations of its properties.

2 Adapting the flexible variety aggregator to NEG

2.1 Utility maximisation

The spatial model developed here assumes an arbitrary finite number of regions, the home region being denoted by subscript i and the j subscript representing any of the foreign regions. Hence, for a flow of manufacturing output $m_{i,j}(h)$, the first subscript indicates the region of production and the second on the region of consumption. Similarly, $\tau_{i,j} \geq 1$ is the iceberg transport cost incurred in shipping the manufacturing good from the home region to the target region. As is standard in the NEG literature, it is assumed that transport within a region is costless, i.e. $\tau_{i,i} = 1$.

Agents are assumed to have the same preferences in all regions, and consume manufacturing goods only. Their utility function is therefore defined over a manufacturing aggregate M_i with price index G_i, and regional expenditure is equal to the wage bill. The overall utility maximisation problem is given by:

$$\begin{cases} \max \quad U_i = M_i \\ \text{s.t.} \quad M_i G_i = w_i L_i \end{cases} \tag{1}$$

Following the methodology developed by Kimball (1995), the manufacturing aggregate M_i is implicity defined by the following integral. $\varphi(x)$ is a sub-utility function that is assumed to be a strictly increasing, concave function, with $\varphi(0) = 0$. The implicit definition of M_i is a particularity of Kimball (1995), however it does not change the discussion of the properties of the sub-utility function $\varphi(x)$ presented in Behrens and Murata (2007). The star subscript in the manufacturing flow indicates that the integral sums all N varieties h consumed in region i, regardless of the region of production:[2]

$$\int_0^N \varphi\left(\frac{m_{*,i}(h)}{M_i}\right) dh = 1 \tag{2}$$

Manufacturing expenditure in region i is simply the integral sum of expenditure per variety, defined over the continuum of varieties.

$$M_i G_i = \int_0^N p_{*,i}(h)\, \tau_{*,i} m_{*,i}(h)\, dh \tag{3}$$

As explained in Fujita et al. (1999), in this kind of NEG model the sub-utility problem can be solved separately from the overall utility maximisation. Indeed, regardless of the amount

[2]The star subscript is introduced in the integral sums and summations to emphasise situations where the summation over varieties or regions does not depend on the origin region of the flow.

of the aggregate M consumed, maximising overall utility involves choosing a composition of varieties within the aggregate that minimises manufacturing expenditure. In this respect the utility problem (1) determines the optimal amount of the manufacturing aggregate regardless of its actual composition. The solution to (1) is trivial:

$$M_i = \frac{w_i L_i}{G_i} \tag{4}$$

It is important to note that this is also the indirect utility function in region i, which will be useful for the welfare analysis in section 3. The separable sub-utility problem involves choosing a combination of manufacturing varieties that minimises the manufacturing expenditure $M_i G_i$, regardless of the overall amount actually spend on manufactures.

$$\begin{cases} \min & \int_0^N p_{*,i}(h)\,\tau_{*,i} m_{*,i}(h)\,dh \\ \text{s.t.} & \int_0^N \varphi\left(\frac{m_{*,i}(h)}{M_i}\right) dh = 1 \end{cases} \tag{5}$$

Using (3) one obtains the following Lagrangian and first order condition:

$$\Lambda_i = \int_0^N p_{*,i}(h)\,\tau_{*,i} m_{*,i}(h)\,dh - \lambda\left(\int_0^N \varphi\left(\frac{m_{*,i}(h)}{M_i}\right) dh - 1\right)$$

$$p_{i,j}(h)\,\tau_{i,j} = \frac{\lambda}{M_j}\varphi'\left(\frac{m_{i,j}(h)}{M_j}\right) \tag{6}$$

The first order condition (6) can be inverted to determine the compensated demand for a variety consumed in region i. In terms of notation, the -1 superscript indicates the inverse function.

$$s_{i,j}(h) = \frac{m_{i,j}(h)}{M_j} = \varphi'^{-1}\left(\frac{p_{i,j}(h)\,\tau_{i,j}}{\tilde{P}_j}\right) \tag{7}$$

Where $s_{i,j}(h)$ is defined as the share of a variety h, produced in region i, in the manufacturing aggregate of a target region j. Equation (7) also assumes the existence of a compositional price index $\tilde{P}_i$:

$$\tilde{P}_i = \frac{\lambda}{M_i} \tag{8}$$

This second price index $\tilde{P}_i$ is a particularity of the Kimball (1995) flexible variety aggregator. It is important to point out that it is generally different from the G_i manufacturing price index of standard Dixit and Stiglitz (1977) NEG models. $\tilde{P}_i$ is used to determine the optimal composition of the manufacturing aggregate. This can be seen in (7), where φ'^{-1}

maps the price of a variety relative to $\tilde{P}_j$ to its share in the aggregate M_j. The manufacturing price index G_i on the other hand, gives the cost of a unit of the manufacturing aggregate, which is visible in equation (3). In the Dixit and Stiglitz (1977) aggregator the manufacturing price index G_i fulfills both of these functions.

Equation (8) defines $\tilde{P}_i$ using the lagrangian multiplier, however once a specific functional form is chosen for $\varphi(x)$, a full specification of this price index can be obtained by replacing the first order condition (6) into the implicit equation (2).[3] The manufacturing price index G_i can be obtained by replacing (7) in the expenditure equation (3). As for $\tilde{P}_i$, a full specification requires choosing a functional form for $\varphi(x)$.

$$G_i = \int_0^N p_{*,i}(h)\,\tau_{*,i}\varphi'^{-1}\left(\frac{p_{*,i}(h)\,\tau_{*,i}}{\tilde{P}_i}\right)dh \tag{9}$$

2.2 Pricing behaviour of firms

Given the demand structure for manufacturing varieties laid out above, it is possible to work out the pricing policy of firms. As is the case in standard NEG models, it is assumed that the production of a manufacturing variety uses only labour, with a fixed cost α and a variable cost β. The total cost of production of a variety h in i is given by:

$$C_i(h) = w_i\left(\alpha + \beta\sum_j m_{i,j}(h)\right) \tag{10}$$

The profit made by a producer in region i on variety h is therefore:

$$\pi_i(h) = \sum_j p_{i,j}(h)\,m_{i,j}(h) - C_i(h) \tag{11}$$

Maximising manufacturing profits with respect to all flows $m_{i,j}(h)$ gives the standard Lerner index first order condition:

$$p_{i,j}(h)\left(\frac{1}{\varepsilon_{i,j}(h)} + 1\right) = \beta w_i \tag{12}$$

Where the inverse price elasticity of demand is given by:

$$\frac{1}{\varepsilon_{i,j}(h)} = \frac{\partial p_{i,j}(h)}{\partial m_{i,j}(h)}\frac{m_{i,j}(h)}{p_{i,j}(h)}$$

[3] This is shown in appendix A.2

The formal definition of the inverse price elasticity of demand is found by taking the derivative of $p_{i,j}(h)$ with respect to $m_{i,j}(h)$ in the first order condition (6).

$$\frac{1}{\varepsilon_{i,j}(h)} = s_{i,j}(h) \frac{\varphi''(s_{i,j}(h))}{\varphi'(s_{i,j}(h))} \tag{13}$$

One can see from this equation that in the Kimball (1995) specification the mark-up of prices on marginal costs depends on the market share $s_{i,j}(h)$ of a variety in the target region. One can also see that the functional form of (13) is simply the Arrow-Pratt measure of relative risk-aversion. This is no coincidence, and is the reason why Behrens and Murata (2007) focus their discussion on the risk aversion properties of their sub-utility functions.

The intuition behind this comes from a standard result of inter-temporal optimisation problems. Blanchard and Fischer (1989) explain that if the overall utility function is additively separable (which is the case of the integral sum in (2)) then the Arrow-Pratt measure of relative risk-aversion is equal to the inverse of the elasticity of substitution.[4] This is the gist of the argument in Behrens and Murata (2007) : if the sub-utility function $\varphi(x)$ exhibits constant relative risk aversion (CRRA), then the elasticity of demand in (13) is also constant, and by construction so are the mark-ups.

If one desires variable elasticities of substitution and variable mark-ups, two avenues are possible, which both require departing from CRRA sub-utility. The first avenue, followed by Behrens and Murata (2007), is to choose $\varphi(x)$ such that it exhibits CARA. The second, suggested by Kimball (1995) and the ensuing new keynesian literature, is to choose $\varphi(x)$ such that it exhibits *variable relative risk aversion*. Although these solutions both lead to variable mark-ups, they implicity involve a tradeoff, which is discussed in the next section.

2.3 The role of the sub-utility function

The functional form chosen for the subutility function $\varphi(x)$ in our VES-NEG model is similar to the one suggested by Dotsey and King (2005). The various derivatives and inverses required to work the model are given in appendix A.1.

$$\varphi(x) = \frac{(\eta x - (\eta - 1))^\rho}{\eta} - \frac{(1-\eta)^\rho}{\eta} \tag{14}$$

One can see from (14) that for $0 < \rho < 1$ and $0 < \eta \leq 1$, $\varphi(x)$ is increasing, concave, and $\varphi(0) = 0$, which satisfies the requirements laid out in section 2.1.

[4]The standard inter-temporal optimisation problems presented in chapters 2 and 6 of Blanchard and Fischer (1989) involve summing the instantaneous utility from the consumption of a single good over continuous time. Here, the dimensions of the problem are inverted, and utility per variety is summed over a continuum of goods at a single point in time. This means that the elasticity of inter-temporal substitution becomes an elasticity of substitution amongst varieties.

We now turn towards discussing how this specification diverges from the CRRA - CES standard of typical Dixit and Stiglitz (1977) models. The discussion will focus on the role of η, which is the parameter of interest. Its role can be understood in two equivalent ways, depending on whether one looks at the curvature or the separability properties of the demand function. Given the specification of $\varphi'^{-1}(x)$ in appendix A.1, the compensated demand for a variety (7) can be expressed as:

$$\frac{m_{j,i}(h)}{M_i} = \frac{1}{\eta}\left(\left(\frac{p_{j,i}(h)\,\tau_{j,i}}{\rho\tilde{P}_i}\right)^{\frac{1}{\rho-1}} + (\eta-1)\right)$$

As pointed out by Dotsey and King (2005), this is the sum of a CES demand and a constant term which depends only on η. In particular, Sbordone (2007) shows that the η parameter controls the curvature of the relative demand curve. Varying η produces a set of demand curves with a wide range of possible curvatures, from convex to concave. Examples of such demand curves are given in the figures of Dotsey and King (2005) and Sbordone (2007).

Examining the separability conditions of Behrens and Murata (2007) gives a second interpretation of the role of η. Indeed, Behrens and Murata show that when the sub-utility function $\varphi(x)$ exhibits CRRA, the demand function $\varphi'^{-1}(x)$ displays the following multiplicative quasi-separablility (MQS) property:

$$\varphi'^{-1}(xy) = \varphi'^{-1}(x) \times f(y)$$

Given the specification of $\varphi'^{-1}(x)$, one can obtain the value of $\varphi'^{-1}(xy)$:

$$\varphi'^{-1}(xy) = \varphi'^{-1}(x) \times \frac{1}{\eta}(y)^{\frac{1}{\rho-1}} + \frac{(\eta-1)}{\eta}$$

As for the previous result, this is the sum of the CRRA - CES solution and a constant parameter which depends only on η. The particularity of the Kimball (1995) aggregator compared to the Behrens and Murata (2007) approach, therefore, is that separability fails to hold in general. This failure nevertheless happens in a very controlled manner: for all values of the arguments x and y, the divergence from MQS is always constant and controlled by η.

It should be apparent from this discussion that for the special case where $\eta = 1$, the divergence term disappears, and the demand function exhibits MQS. This implies that the entire model reverts to the CRRA - CES case. Additionally, specifying $\rho = (\sigma-1)/\sigma$ recovers the generic Dixit and Stiglitz (1977) benchmark with constant elasticity of substitution σ, which is the standard model of NEG.

The tradeoff mentioned in the previous section is that the VES model developed here involves relaxing the assumption of separability, while the Behrens and Murata (2007) CARA solution does not. The upshot, however, is that a NEG model based on the Kimball (1995) aggregator will nest the Dixit and Stiglitz (1977) benchmark as a special case ($\eta = 1$), which is not true of the model proposed by Behrens and Murata. Indeed, the use of two entirely different sub-utility functions means that a direct comparison of the two model predictions is not possible. A simple illustration of this is the fact that CARA consumers become asymptotically satiated, whereas CRRA ones do not. The Behrens and Murata CARA case uses an exponential sub-utility of the form:

$$u(x) = k - \kappa e^{-ax}$$

Clearly $\lim_{x \to \infty} u(x) = k$. The CRRA case uses a sub-utility function similar to (14) with $\eta = 1$, and $\lim_{x \to \infty} \varphi(x) = \infty$. This means that the underlying preference structure of consumers is radically different, making comparisons difficult between the CES and VES cases.

2.4 Solving for prices, quantities and mark-ups

The choice of specification for $\varphi(x)$ allows the determination of all the price indexes defined above. Starting with the compositional price index $\tilde{P}_i$, replacing (7) into the implicit definition of the aggregator (2) allows the definition of $\tilde{P}_i$ as a price index rather than the initial Lagrange multiplier based definition in equation (8). The details of this are given in appendix A.2.

$$\tilde{P}_i = \frac{\left(\int_0^N (p_{*,i}(h)\,\tau_{*,i})^{\frac{\rho}{\rho-1}}\, dh \right)^{\frac{\rho-1}{\rho}}}{\rho\left(\eta + N(1-\eta)^{\rho}\right)^{\frac{\rho-1}{\rho}}} \tag{15}$$

Once $\tilde{P}_i$ is specified, it is possible to work out the manufacturing share in equation (7). Additionally, one can retrieve the output flows $m_{i,j}(h)$ by multiplying the share (7) by the manufacturing aggregate M_j. Using the result obtained above for $\tilde{P}_i$ and the general specification of $\varphi(x)$ in appendix equation (A-3), one can obtain the following:

$$s_{i,j}(h) = \omega_1 \frac{(p_{i,j}(h)\,\tau_{i,j})^{\frac{1}{\rho-1}}}{\left(\int_0^N (p_{*,j}(h)\,\tau_{*,j})^{\frac{\rho}{\rho-1}}\, dh \right)^{\frac{1}{\rho}}} + \omega_2 \tag{16}$$

Here ω_1 and ω_2 are parameter bundles that are introduced to clarify the notation.[5] Note that $\omega_1 = 1$ and $\omega_2 = 0$ when $\eta = 1$, and so disappear from expression (16) in that case.

$$\omega_1 = \frac{\left(\eta + N\left(1-\eta\right)^{\rho}\right)^{\frac{1}{\rho}}}{\eta} \quad \text{and} \quad \omega_2 = \frac{\eta - 1}{\eta}$$

Next, the manufacturing price index G_i can be calculated by inserting (16) into the specification given by (9). As for previous results, the detailed derivation of this specification is available in appendix A.2. Importantly, one can again see in the equation below the confirmation that with $\eta = 1$ and $\rho = (\sigma - 1)/\sigma$, the price index of manufactures G_i reverts to the typical Dixit and Stiglitz (1977) structure found in CES NEG models.

$$G_i = \omega_1 \left(\int_0^N \left(p_{*,i}\left(h\right)\tau_{*,i}\right)^{\frac{\rho}{\rho-1}} dh \right)^{\frac{\rho-1}{\rho}} + \omega_2 \left(\int_0^N p_{*,i}\left(h\right)\tau_{*,i} dh \right) \tag{17}$$

Finally, one can use the specification of the sub-utility function to obtain the elasticity of demand in equation (13), and therefore the mark-up of prices over marginal costs. Given the sub-utility function described in equation (14) and using $\rho = (\sigma - 1)/\sigma$, the elasticity of demand for a variety h in a region is:

$$\varepsilon_{i,j}\left(h\right) = -\sigma \left(1 - \frac{\omega_2}{s_{i,j}\left(h\right)}\right) \tag{18}$$

Setting the divergence parameter η equal to one, one retrieves the standard Dixit and Stiglitz (1977) elasticity σ. This implies the following mark-up equation:

$$p_{i,j}\left(h\right) = \frac{s_{i,j}\left(h\right) - \omega_2}{s_{i,j}\left(h\right)\rho - \omega_2}\beta w_i \tag{19}$$

Taking the derivative of the price with respect to the market share of a variety, one can see that for the parameter values specified in section 2.3 ($0 < \rho < 1$ and $0 < \eta \leq 1$) there exists a pro-competitive effect:

$$\frac{\partial p_{i,j}\left(h\right)}{\partial s_{i,j}\left(h\right)} = \frac{\omega_2\left(\rho - 1\right)}{\left(s_{i,j}\left(h\right)\rho - \omega_2\right)^2}\beta w_i \geq 0$$

If the market share of a variety in a region falls (for example because of an increase in varieties available), the mark-up will be lower. As for the previous equations, one can see that assuming $\eta = 1$ and $\rho = (\sigma - 1)/\sigma$ gives the standard CES fixed mark-up Dixit and Stiglitz (1977) result. Furthermore, examination of equation (19) reveals that as for Behrens

[5] One can see here that ω_2 is in fact the divergence term presented in the discussion on separability in section 2.3. It therefore plays a crucial role in the model, as it controls the divergence from the CES specification.

and Murata (2007) the model has a competitive limit. Indeed, one can see from (19) that:

$$\lim_{s_{i,j}(h)\to 0} p_{i,j}(h) = \lim_{s_{i,j}(h)\to 0} \frac{s_{i,j}(h) - \omega_2}{s_{i,j}(h)\rho - \omega_2}\beta w_i = \beta w_i$$

Thus, as the share of each variety $s_{i,j}(h)$ tends to zero the price tends to the marginal cost.

3 A variable elasticity of substitution NEG model

3.1 Closing the model and describing equilibrium

In order to close the proposed NEG model, factor markets need to be introduced. As is standard in the literature and visible in equation (10), labour is the only factor, and in the interest of simplicity, it is assumed in the following that labour cannot migrate between regions. This is a valid assumption for both the cases examined below: it is sensible in the autarchic case, and does not affect the properties of the free trade equilibrium, as factor prices will be shown to equalise regardless of whether migration occurs or not. The labour market clears in all regions, so that the the amount of labour available in each region L_i equals the sum of the input requirements in that region.

$$L_i = \int_0^N \left(\alpha + \beta \sum_j m_{i,j}(h)\right) dh \tag{20}$$

Within this context, an equilibrium is a set of prices $p_{i,j}(h)$, variety shares $s_{i,j}(h)$, wages $w_i(h)$ and firm masses $n_i(h)$ for which the labour markets clear and profits (11) are zero. Proposition 1 below shows that an equilibrium will always be symmetric within regions, in other words, that varieties produced in the same region will have the same pricing patterns and output flows.

Proposition 1: *For any two varieties h and k produced in region i, $p_{i,j}(h) = p_{i,j}(k) = p_{i,j}$ for all transport costs $\tau_{i,j}$. This implies that $m_{i,j}(h) = m_{i,j}(k) = m_{i,j}$.*

Proof: See appendix B.

An important aspect visible from the pricing equation (12) is that the price that a firm will charge customers in a given region depends on the price elasticity of demand in that region. This is not a problem in a standard CES model, as this elasticity is constant and firms charge the same mill price to all consumers. In our VES model, this is not necessarily the case as the elasticity of demand is variable. Indeed, Proposition 2 establishes that as a general rule, unless iceberg transport costs are absent, firms cannot practice mill pricing, and pricing to market occurs.

Proposition 2: *If $\tau_{i,j} > 1$ $\forall i \neq j$, firms charge different prices for different regional flows so $p_{i,i} \neq p_{i,j}$. A mill-pricing equilibrium $p_{i,i} = p_{i,j}$ is only possible if $\tau_{i,j} = 1$ $\forall i, j$.*

Proof: See appendix C.

Note that Proposition 2 does not investigate whether mill pricing is the *only* equilibrium in the absence of transport costs. Rather, it shows that in the absence of transport costs a mill pricing equilibrium exists. It is reasonable to assume that firms do choose mill pricing in the when transport is costless, as all the regional markets regardless of their location effectively become part of the local market.

3.2 The autarchic equilibrium and optimal firm mass

The autarchic equilibrium occurs when the transport costs between any two regions i and j tend to infinity, i.e. $\tau_{i,j} \to \infty$. As is to be expected intuitively, one can see from the price index $\tilde{P}_i$ in (15) that all varieties not produced within the region are removed from the index. Similarly, one can see from (16) that the share variable for these varieties will reduce to $s_{i,j} = \omega_2$. Examining the mark-up specification (19) shows that the price of these flows will be equal to zero. The implication is that only the regional variables are important in defining the equilibrium. Their equilibrium values are denoted with a superscript a.

From the zero profit condition (11) in region i, one can obtain the equilibrium quantities produced within region i:

$$m^a_{i,i} = \frac{1}{\beta}\left(\frac{L_i}{n^a_i} - \alpha\right) \tag{21}$$

Next, replacing the modified budget constraint (3) in region i, $m^a_{i,i} = w_i L_i / n_i p_i$, and the price equation (12) into the labour market clearing condition (11), gives the following expression for the equilibrium firm mass and elasticity of demand

$$n^a_i = -\frac{L_i}{\alpha \varepsilon^a_{i,i}} \tag{22}$$

$$\varepsilon^a_{i,i} = -\sigma\left(1 - \frac{\omega_2}{s^a_{i,i}}\right) \tag{23}$$

Next, using (16) and the definition of ω_1,[6], taking into account varieties not produced in the region disappear from the integral sums, one obtains the share variable in autarchy:

$$s^a_{i,i} = \left(\frac{\eta^{1-\rho}}{n^a_i} + (-\omega_2)^\rho\right)^{\frac{1}{\rho}} + \omega_2 \tag{24}$$

As explained above, if the divergence parameter η is set to one, $\omega_2 = 0$, and the equilibrium firm mass (22) reverts to the standard CES Dixit-Stiglitz result, $n^a_i = L_i/\alpha\sigma$. In a VES setting with $\omega_2 < 0$, solving explicitly for the firm mass can be difficult, as isolating n^a_i in equation (24) is complicated. However, Proposition 3 below show that $n^a_{i,i}$ always exists, which means it can be solved for it numerically.

Proposition 3: *In an autarchic setting, there always exists a unique firm mass n^a_i which is a solution to equations (22), (23) and (24).*

Proof: see appendix D.

Furthermore, observing equations (22) and (23), one can immediately see that in the VES case, with $\omega_2 < 0$, the elasticity of demand is greater in absolute value than in the CES case and the firm mass will be lower. Figure 1 confirms this: under VES preferences, the equilibrium number of firms is lower than under CES preferences. When entry occurs the protection of market power offered by the increased product differentiation does not completely offset the increased competition. This is visible through the increase in the elasticity of demand, which reduces the mark-ups. It is therefore intuitive that less firms survive in equilibrium compared to a situation where mark-ups remain constant.

The next step is to examine the optimality of this equilibrium. A well known result of Dixit-Stiglitz based NEG models, pointed out by Behrens and Murata (2006) in an extension to their basic model, is that the equilibrium number of varieties is always optimal. We show that this is not the case with VES preferences, by showing that the the number of varieties which maximises welfare, which we denote n^o_i is not equal to the equilibrium n^a_i. As pointed out in section 2.1, the manufacturing aggregate M_i is also the measure of welfare in i. Inverting (7) gives us a specification of M_i that takes into account the resource constraint embodied in the equilibrium quantity (21).

$$M_i\left(n^o_i\right) = \frac{m^o_{i,i}}{s^o_{i,i}} = \frac{\frac{1}{\beta}\left(\frac{L_i}{n^o_i} - \alpha\right)}{\left(\frac{\eta^{1-\rho}}{n^o_i} + (-\omega_2)^\rho\right)^{\frac{1}{\rho}} + \omega_2} \tag{25}$$

[6] In autarchy, ω_1 depends on the number of local varieties and not the overall number of varieties N. This can be shown by recalculating $\tilde{P}_i$ for the autarchic case

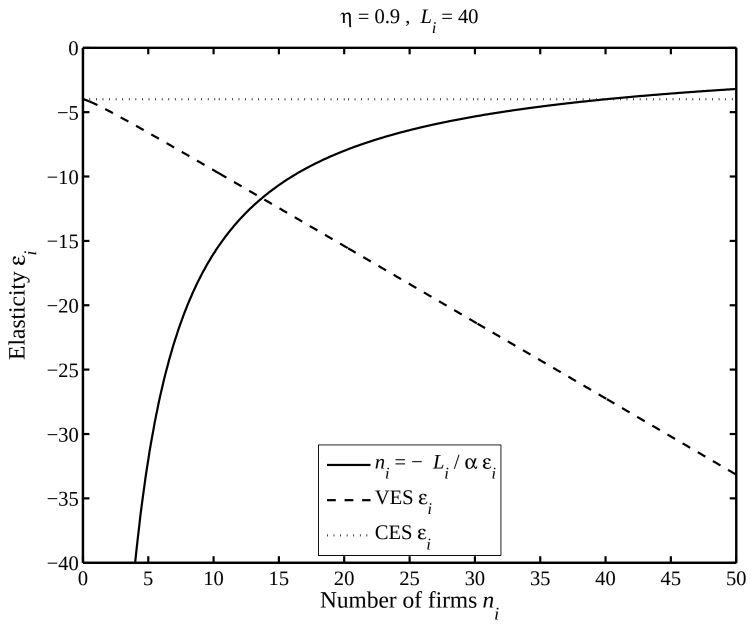

Figure 1: Equilibrium firm mass, CES vs. VES

Taking the first order condition $M_i(n_i^o)$ and setting it equal to zero gives the following expression: [7]

$$\frac{1}{n_i^o} = \left(\frac{1}{n_i^a} + \frac{(\sigma - 1)(-\omega_2)^{\rho}}{\eta^{1-\rho}} \right) \frac{s_{i,i}^o}{s_{i,i}^o - \sigma \omega_2} \tag{26}$$

One can see from (26) that $n_i^o = n_i^a$ occurs if and only if $\omega_2 = 0$, in other words if preferences are CES. Under the VES specification developed here, with $\eta < 1$, the equilibrium number of varieties n_i^a is not optimal. The sign of the deviation is not clear from (26), but numerical analysis shown in Figure 2 reveals that $n_i^o < n_i^a$. This is comparable to the findings of Behrens and Murata (2006). As they point out, the entry of an extra firm imposes a negative externality on firms that are already in the market, through a reduction in mark-ups. As a result of this negative externality, excess entry occurs compared to the socially optimal level of varieties.

An important result of this analysis is that the autarchic equilibrium displays the same qualitative properties as the autarchic CARA model developed in Behrens and Murata (2006), in particular the existence of excess entry. As was pointed out in the previous sections, the main difference with the CARA model of Behrens and Murata is the fact that the Kimball aggregator nests the standard CES specification. As a result, it is much easier to compare the predictions of the model to the benchmark.

[7] Details on this first order condition are given in appendix A.3

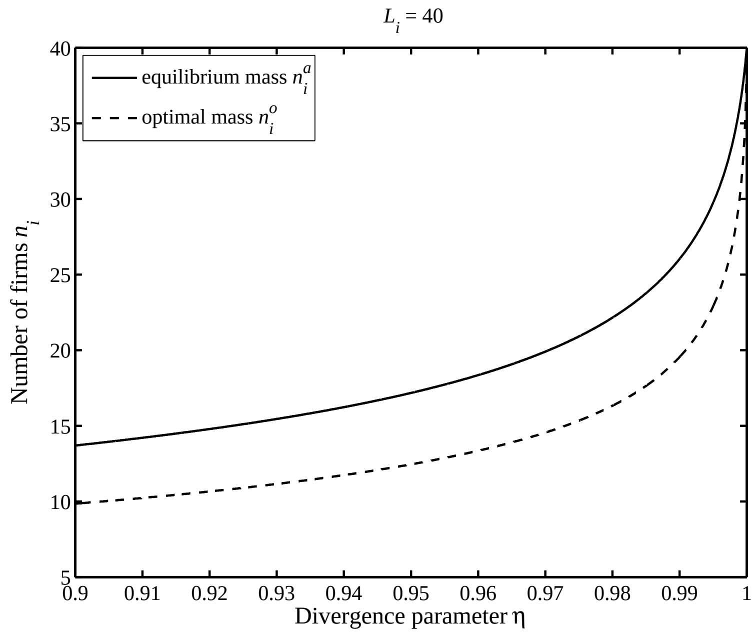

Figure 2: Equilibrium vs. optimal firm mass

3.3 Free trade and efficiency

After analysing the autarchic equilibrium, we identify the gains from trade by investigating the free trade free equilibrium, when there are no transport costs between regions i.e. $\tau_{i,j} = 1$ $\forall i, j$. As explained above, it is assumed that under these transport cost conditions, firms choose the mill pricing equilibrium. Proposition 4 shows that this leads prices to equalise over regions and varieties, which also implies a unique wage rate over regions.

Proposition 4: *In the absence of transport costs (i.e. $\tau_{i,j} = 1 \ \forall \ i, j$), there exists a symmetric equilibrium, where across regions there is: a unique price p for all varieties, a unique share variable s, a unique price index G and a unique wage w.*

Proof: see appendix E.

Because prices and wages equalise over regions, the zero profit condition is the same for all firms. We define $L = \sum_i L_i$ as the overall amount of labour, and as previously, $N = \sum_i n_i$ is the overall firm mass. The equalisation of the share variables s implies that in free trade firms produce the same aggregate amount over regions:

$$m^f = \sum_j m_{i,j} = s \sum_j M_j$$

Furthermore, one can see from equation (4) that Proposition 4 implies that per-capita welfare equalises over regions, so that $M_i/M_j = L_i/L_j$. This means that any given output flow can be worked out simply from the aggregate output and the labour share:

$$m_{i,j} = \frac{m^f L_j}{L} \quad \forall i, j$$

Using this formula, the budget constraint (3) can be expressed as $N^f pm^f = wL$. Using this, the price equation (12), the labour market clearing condition (20) and the zero profit equation (11) gives the following solutions for aggregate firm output and firm mass:

$$m^f = \frac{1}{\beta}\left(\frac{L}{N^f} - \alpha\right) \tag{27}$$

$$n_i^f = -\frac{L_i}{\alpha\varepsilon} \tag{28}$$

Summing over regions, the total amount of varieties in free trade is therefore:

$$N^f = \sum_i n_i^f = -\frac{L}{\alpha\varepsilon} \tag{29}$$

With the unique price elasticity given by:

$$\varepsilon = -\sigma\left(1 - \frac{\omega_2}{s}\right) \tag{30}$$

And the following share variable:

$$s = \left(\frac{\eta^{1-\rho}}{N^f} + (-\omega_2)^\rho\right)^{\frac{1}{\rho}} + \omega_2 \tag{31}$$

Because the functional form for the system of equations (29) - (31) describing the total number of varieties under free trade N^f is the same as (22) - (24) for n_i^a in the autarchic case, Proposition 3 holds. This means that this system again has a unique solution N^f. The system can therefore be solved numerically, using exactly the same approach as for the autarchic case.

Figure 3 shows the equilibrium mass of firms as a function of the amount of labour available, both for the CES and VES specifications. It can be read in two ways: either as the autarchic firm mass n_i^a given regional labour L_i, or as the total free-trade firm mass N^f given total labour L. As a result this figure illustrates the effect of free trade on the equilibrium mass of firms in each region. For the purpose of illustration, we assume two identical regions with an endowment of 40 units of labour. Figure 3 confirms the CES result that firm mass n_i does not change as a result of trade. For the VES model, however, the

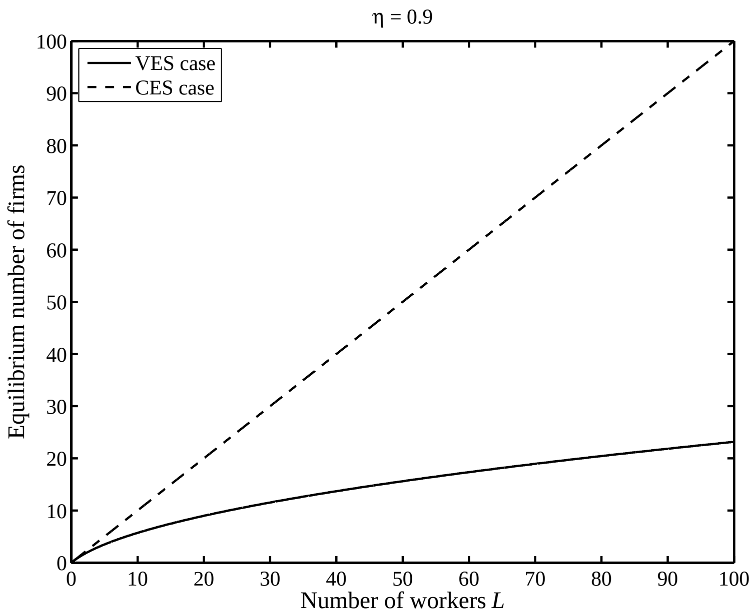

Figure 3: Equilibrium firm mass as a function of labour

figure shows that in autarky each region has 14 varieties while the total number of varieties in free trade is just above 20. Given that in free trade $n_i = (L_i/L) \times N$, the number of varieties produced in each region falls, even though consumers can access more varieties overall. This means that the smaller number of firms in each region, taking better advantage of the increasing returns to scale exhibited by the manufacturing sector in (10).

As for the autarchic case, the equilibrium firm mass can be compared to the optimal firm mass. Because of the existence of a symmetric equilibrium, the equations describing the aggregate welfare under free trade are the same as the ones describing the welfare of a region in autarky. Therefore, the following relation exists between the optimal level of firms N^o and the equilibrium level of firms N^f:

$$\frac{1}{N^o} = \left(\frac{1}{N^f} + \frac{(\sigma - 1)(-\omega_2)^\rho}{\eta^{1-\rho}} \right) \frac{s}{s - \sigma\omega_2}$$

Figure 4 replicates the analysis of Figure 2 with twice the amount of labour, completing the illustration of free trade between two identical regions. One can see in Figure 4 that the shortfall between N^o and N^f is slightly larger in absolute terms than the one displayed for the autarky case in Figure 2. However, as for Figure 3, relative to the greater number of regions and the larger population, one can see that free trade brings an improvement in efficiency by bringing the overall equilibrium allocation closer to the optimal one. This is consistent with the competitive limit on mark-ups shown in section 2.

As for the autarky benchmark, these qualitative predictions under free trade are the

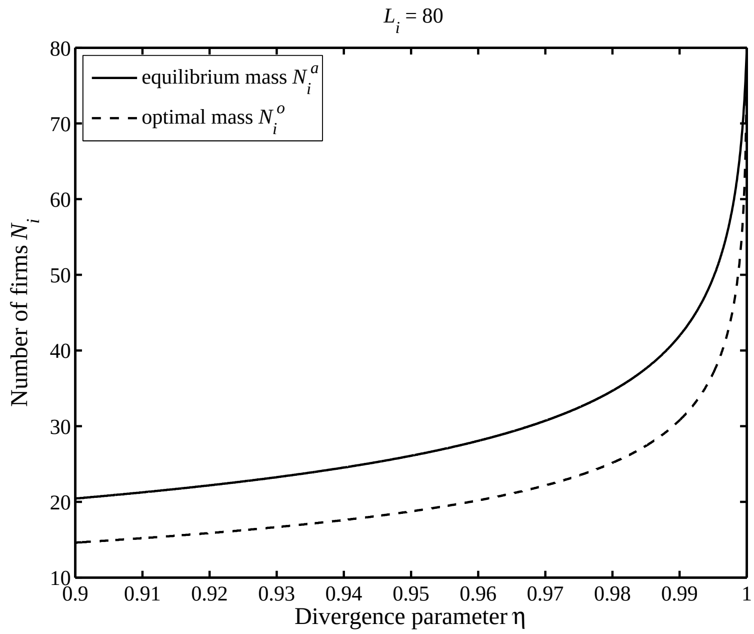

Figure 4: Equilibrium vs. optimal firm mass

same as the ones made in Behrens and Murata (2006). This is particularly true of the fact that free trade increases the available varieties while reducing the number of local varieties, which Behrens and Murata report as being a standard finding in the literature. However, as for the autarky case, the VES results have the added advantage of being directly comparable to the standard CES benchmark.

3.4 Applications

The central application suggested for this VES model is the development of extended NEG or new trade theory models that account for pro-competitive effects and variable mark-ups while still remaining comparable to the CES original versions. The CARA approach suggested by Behrens and Murata (2007) has the advantage of providing a tractable method of investigation, by retaining some form of separability in the demand function. The cost, however, is that the closed form solutions obtained through this method are not easily compared with the CES benchmark.

This is a crucial point, as the the empirical testing of NEG predictions is an important issue, and most of these predictions are formulated using the CES Dixit and Stiglitz (1977) model. A rigourous approach to the empirical testing of these predictions hence requires a theoretical model that contains the Dixit-Stiglitz structure, but accounts for the spatial variations in market power that are bound to exist in the data. While the model developed here might be able do this, we argue that the Behrens and Murata (2007) model cannot, precisely because of the lack of direct comparison with the CES benchmark.

From the specific point of view of NEG, there are therefore further empirical applications for the VES model developed here. Indeed, as has been made clear throughout the discussion, the transition from the CES benchmark to a VES model is controlled by the single divergence parameter η. As explained in section 2.3, η is linked to the curvature of the demand curves, which means that one could expect to capture it by analysing the spatial structure of local demand structures. This would then allow for testing of core NEG predictions, controlling for the fact that preferences diverge from the CES case by a factor η.

A promising avenue in that respect is the use of the spatial structure of firm mark-ups. Studies such as Siotis (2003) on Spain or Konings et al. (2005) on Romania and Bulgaria show that it is possible to use firm data to estimate average mark-ups, using the methodology developed by Hall (1988) and extended by Roeger (1995). Using geo-coded firm data, this would provide the empirical data on the spatial structure of mark-ups necessary to test within a given country the NEG model developed here. Another related possibility is testing the pricing to market behaviour predicted within the model using trade data between countries. While this requires modifying the model to account for exchange rate effects, recent work by Gust et al. (2006) shows that one can estimate a model of export pricing based on the Kimball (1995) aggregator and identify the deviation of the demand structures from the CES benchmark.

4 Conclusion

This paper evaluates the approach initially proposed by Behrens and Murata (2007) and extended in Behrens and Murata (2006) to address the issue lack of pro-competitive effects in the standard Dixit and Stiglitz (1977) CES setting. Although the reasons behind the presence these fixed mark-ups in a CES setting are well explained by Behrens and Murata, the solution they suggest ignores pre-existing literature that has successfully developed mechanisms to combine product differentiation and the pro-competitive effect of firm entry. Furthermore we argue that while the CARA solution they advocate is technically correct, it is also somewhat unsatisfactory. The following arguments are probably valid for most applications of monopolistically competitive models, but particularly so for NEG, a field practically entirely grounded on the Dixit-Stiglitz CES model.

Both the Behrens and Murata (2007) approach and the one suggested here generate variable mark-ups, with a competitive limit. Both exhibit an equilibrium number of varieties that is above the optimal level, with this disparity being reduced under free trade. In order to do so, however, the Behrens and Murata CARA model requires an entirely different sub-utility function, while the Kimball (1995) approach only requires a change in parameter within the same sub-utility structure. This implies that the CARA and CRRA models in Behrens and Murata (2007) are not directly comparable. By contrast, the fact that the Kimball (1995) aggregator encompasses both cases within the same structure makes it a more general approach, which we suggest can be of major interest in attempting to assess empirically the predictions of the standard CES models of NEG.

References

Behrens, K., Murata, Y. (2006), Gains from trade and efficiency under monopolistic competition: a variable elasticity case, *CORE Discussion paper*, 2006/49.

— (2007), General equilibrium models of monopolistic competition, *Journal of Economic Theory*, 136: 776–787.

Blanchard, O., Fischer, S. (1989), *Lectures in Macroeconomics*, MIT Press, Cambridge, MA.

Calvo, G. (1983), Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, 12: 383–398.

Dixit, A.K., Stiglitz, J.E. (1977), Monopolistic competition and optimum product diversity, *American Economic Review*, 67: 297–308.

Dotsey, M., King, R.G. (2005), Implications of state-dependent pricing for dynamic macroeconomic models, *Journal of Monetary Economics*, 52: 213–242.

Fujita, M., Krugman, P., Venables, A.J. (1999), *The Spatial Economy : Cities, regions and international trade*, MIT Press, Cambridge, MA.

Gust, C.J., Leduc, S., Vigfusson, R. (2006), Trade integration, competition, and the decline in exchange-rate pass-through, *Federal Reserve Board International Finance Discussion Paper*, 864.

Hall, R.E. (1988), The relation between price and marginal cost in u.s. industry, *The Journal of Political Economy*, 96: 921–947.

Kimball, M.S. (1995), The quantitative analytics of the basic neomonetarist model, *Journal of Money, Credit and Banking*, 27: 1241–1277.

Konings, J., Van Cayseele, P., Warzynski, F. (2005), The effects of privatization and competitive pressure on firms' price-cost margins: Micro evidence from emerging economies, *Review of Economics and Statistics*, 87: 124–134.

Krugman, P. (1998), Space: The final frontier, *Journal of Economic Perspectives*, 12: 161–174.

Melitz, M.J., Ottaviano, G.I.P. (2008), Market size, trade, and productivity, *Review of Economic Studies*, 75: 295–316.

Ottaviano, G.I.P., Tabuchi, T., Thisse, J.F. (2002), Agglomeration and trade revisited, *International Economic Review*, 43: 409–436.

Roeger, W. (1995), Can imperfect competition explain the difference between primal and dual productivity measures? estimates for u.s. manufacturing, *The Journal of Political Economy*, 103: 316–330.

Sbordone, A.M. (2007), Globalization and inflation dynamics: the impact of increased competition, *NBER Working paper*, (13556).

Siotis, G. (2003), Competitive pressure and economic integration: an illustration for spain, 1983-1996, *International Journal of Industrial Organization*, 21: 1435–1459.

A Mathematical appendix

A.1 Sub-utility function

Given the chosen specification for the sub-utility function (14), the following specifications are used in the determination of the model.

First derivative: $\varphi'(x) > 0$ for $0 < \rho < 1$ and $0 < \eta \leq 1$

$$\varphi'(x) = \rho\,(\eta x - (\eta - 1))^{\rho - 1} \tag{A-1}$$

Second derivative: $\varphi''(x) < 0$ for $0 < \rho < 1$ and $0 < \eta \leq 1$

$$\varphi''(x) = \eta\rho\,(\rho - 1)\,(\eta x - (\eta - 1))^{\rho - 2} \tag{A-2}$$

Inverse of the first derivative:

$$\varphi'^{-1}(x) = \frac{1}{\eta}\left(\left(\frac{x}{\rho}\right)^{\frac{1}{\rho-1}} + (\eta - 1)\right) \tag{A-3}$$

A.2 Price indexes

The compositional index $\tilde{P}_i$ of the Kimball (1995) flexible variety aggregator is initially defined simply as a modification of the Lagrange multiplier in (8). By re-inserting the first order condition (7) into (2) which implicity defines M_i, one can show that $\tilde{P}_i$ is indeed a price index.

$$\int_0^N \varphi\left(\varphi'^{-1}\left(\frac{p_{*,i}(h)\,\tau_{*,i}}{\tilde{P}_i}\right)\right) dh = 1 \tag{A-4}$$

Given the specifications of $\varphi(x)$ given in (14) and (A-3), one can work out the integral in the previous equation.

$$\int_0^N \left(\frac{p_{*,i}(h)\,\tau_{*,i}}{\tilde{P}_i}\right)^{\frac{\rho}{\rho-1}} dh = \rho^{\frac{\rho}{\rho-1}}\left(\eta + N\,(1-\eta)^{\rho}\right)$$

$$\tilde{P}_i = \frac{\left(\int_0^N (p_{*,i}(h)\,\tau_{*,i})^{\frac{\rho}{\rho-1}}\, dh\right)^{\frac{\rho-1}{\rho}}}{\rho\left(\eta + N\,(1-\eta)^{\rho}\right)^{\frac{\rho-1}{\rho}}} \tag{A-5}$$

From this, one can obtain for each variety the $p_{*,i}(h)\tau_{*,i}/\tilde{P}_i$ ratio, which can then be used to determine manufacturing price index G_i. This ratio can be used to derive the specification of G_i, given in equation (9) in terms of $\varphi(x)$ and the price to price index ratio defined above.

$$G_i = \int_0^N p_{*,i}(h)\,\tau_{*,i}\varphi'^{-1}\left(\frac{p_{*,i}(h)\,\tau_{*,i}}{\tilde{P}_i}\right)dh$$

The $p_{*,i}(h)\tau_{*,i}/\tilde{P}_i$ price ratio and the functional form of φ'^{-1} in (A-3) can be used to replace the $\varphi'^{-1}(p_{*,i}(h)\tau_{*,i}/\tilde{P}_i)$ notation. Factoring the constant terms out of the integral sum and rearranging gives the specification of the manufacturing price index. One can see that setting $\eta = 1$ not only results in the standard Dixit and Stiglitz (1977) CES price index for G_i, but also for the compositional price index $\tilde{P}_i$ in (A-5).

$$G_i = \frac{(\eta + N(1-\eta)^\rho)^{\frac{1}{\rho}}}{\eta}\left(\int_0^N (p_{*,i}(h)\,\tau_{*,i})^{\frac{\rho}{\rho-1}}\,dh\right)^{\frac{\rho-1}{\rho}} + \frac{\eta-1}{\eta}\int_0^N p_{*,i}(h)\,\tau_{*,i}dh \qquad \text{(A-6)}$$

A.3 Optimal firm mass

Equation (25) gives the welfare as a function of firm mass $M_i(n_i) = m_{i,i}/s_{i,i}$. The socially optimal number of firms, n_i^o, is given by the first order condition on welfare $dM_i(n_i^o)/dn_i^o = 0$.

$$\frac{dM_i(n_i^o)}{dn_i^o} = \frac{\frac{dm_{i,i}^o}{dn_i^o}s_{i,i}^o - \frac{ds_{i,i}^o}{dn_i^o}m_{i,i}^o}{\left(s_{i,i}^o\right)^2} = 0$$

Given the specification of $m_{i,i}$ in (21) and $s_{i,i}$ in (24), the first order condition is:

$$\left(-L_i s_{i,i}^o + \frac{\eta^{1-\rho}}{\rho}\left(\frac{L_i}{n_i^o} - \alpha\right)\left(\frac{\eta^{1-\rho}}{n_i^o} + (-\omega_2)^\rho\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{\beta\,(n_i^o)^2\left(s_{i,i}^o\right)^2} = 0$$

The zero value in the first order condition must come from the term in brackets. Rearranging this term by isolating n_i^o on the left hand side gives

$$\frac{1}{n_i^o} = \frac{\alpha}{L_i} + \frac{\rho s_{i,i}^o}{\eta^{1-\rho}\left(s_{i,i}^o - \omega_2\right)^{1-\rho}}$$

Introducing $\rho = (\sigma-1)/\sigma$ and the definition of the elasticity of demand in (18):

$$\frac{1}{n_i^o} = \frac{\alpha}{L_i} - \frac{\sigma-1}{\varepsilon_{i,i}^o\eta^{1-\rho}}\left(s_{i,i}^o - \omega_2\right)^\rho$$

Using to the definition of the share variable (24) to substitute the $(s_{i,i}^o - \omega_2)^\rho$ term:

$$\frac{1}{n_i^o} = \frac{\alpha}{L_i} - \frac{\sigma - 1}{\varepsilon_{i,i}^o \eta^{1-\rho}} \left(\frac{\eta^{1-\rho}}{n_i^o} + (-\omega_2)^\rho \right)$$

Finally, solving for $1/n_i^o$ gives:

$$\frac{1}{n_i^o} = \left(\frac{-\alpha \varepsilon_{i,i}^o}{L_i} + \frac{(\sigma - 1)(-\omega_2)^\rho}{\eta^{1-\rho}} \right) \frac{1}{1 - \left(\varepsilon_{i,i}^o + \sigma\right)}$$

Using (22) to introduce n_i^a and (23) to substitute for the elasticity $\varepsilon_{i,i}^a$ in the denominator of the right hand side the gives the specification used in section 3.2.

B Proof of Proposition 1 (Symmetric pricing of regional varieties)

Proposition 1: *For any two varieties h and k produced in region i, $p_{i,j}(h) = p_{i,j}(k) = p_{i,j}$ for all transport costs $\tau_{i,j}$*

Proof:

Combining the first order conditions for profit maximisation for the varieties h and k (12) and the definition of the inverse elasticity of demand (13) gives :

$$\begin{cases} p_{i,j}(h) \left(s_{i,j}(h) \frac{\varphi''(s_{i,j}(h))}{\varphi'(s_{i,j}(h))} + 1 \right) = \beta w_i \\ p_{i,j}(k) \left(s_{i,j}(k) \frac{\varphi''(s_{i,j}(k))}{\varphi'(s_{i,j}(k))} + 1 \right) = \beta w_i \end{cases} \tag{A-7}$$

With

$$\begin{cases} s_{i,j}(h) = \varphi'^{-1} \left(\frac{p_{i,j}(h) \tau_{i,j}}{\tilde{P}_j} \right) \\ s_{i,j}(k) = \varphi'^{-1} \left(\frac{p_{i,j}(k) \tau_{i,j}}{\tilde{P}_j} \right) \end{cases} \tag{A-8}$$

The right hand side of the conditions in (A-7) are equal, so the following must hold:

$$p_{i,j}(h) \left(s_{i,j}(h) \frac{\varphi''(s_{i,j}(h))}{\varphi'(s_{i,j}(h))} + 1 \right) = p_{i,j}(k) \left(s_{i,j}(k) \frac{\varphi''(s_{i,j}(k))}{\varphi'(s_{i,j}(k))} + 1 \right) \tag{A-9}$$

Furthermore, both the compositional price index $\tilde{P}_j$ and transport costs $\tau_{i,j}$ are given to the producers of varieties h and k in region i. Therefore, the left hand side of (A-9) is a function of $p_{i,j}(h)$ only, and the right hand side is the same function of $p_{i,j}(k)$ only. Relation (A-9) can only hold if $p_{i,j}(h) = p_{i,j}(k)$. This implies that in (A-8) $s_{i,j}(h) = s_{i,j}(k)$, and from the definition of $s_{i,j}(h)$ in (7), one infers $m_{i,j}(h) = m_{i,j}(k)$. ■

C Proof of Proposition 2 (Pricing to market)

Proposition 2: *If $\tau_{i,j} > 1\ \forall i \neq j$, firms charge different prices for different regional flows so $p_{i,i} \neq p_{i,j}$. A mill-pricing equilibrium $p_{i,i} = p_{i,j}$ is only possible if $\tau_{i,j} = 1\ \forall i, j$.*

Proof:

From the first order condition (12) one can see that for firms to charge the same price to all regions requires that the elasticity of demand for that variety to be equal over regions. From the specification of elasticities in (13), this also implies equalisation of the shares:

$$p_{i,i} = p_{i,j} \quad \Leftrightarrow \quad \varepsilon_{i,i} = \varepsilon_{i,j} \quad \Leftrightarrow \quad s_{i,i} = s_{i,j}$$

Assume that firms choose mill pricing $p_{i,j} = p_{i,i}$, so that the share variables (16) can be expressed solely in terms of the prices charged in the region of production:

$$\begin{cases} s_{i,i} = \omega_1 \dfrac{(p_{i,i})^{\frac{1}{\rho-1}}}{\sum\limits_{*} \left(n_* (p_{*,*})^{\frac{\rho}{\rho-1}} (\tau_{*,i})^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} + \omega_2 \\ s_{i,j} = \omega_1 \dfrac{(p_{i,i})^{\frac{1}{\rho-1}} (\tau_{i,j})^{\frac{1}{\rho-1}}}{\sum\limits_{*} \left(n_* (p_{*,*})^{\frac{\rho}{\rho-1}} (\tau_{*,j})^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} + \omega_2 \end{cases} \tag{A-10}$$

There are three possible cases depending on the value of transport costs τ:

1. In general for all origin regions shipping costs equalise for all target regions $\tau_{*,i} = \tau_{*,j}$. Because the home region is also a target region and by definition $\tau_{i,i} = 1$, this implies $\tau_{i,j} = 1\ \forall i, j$. Then the numerators and denominators in (A-10) equalise, and $s_{i,i} = s_{i,j}$. This supports the mill-pricing equilibrium.
2. The transport cost structure is such that varying values of the numerators in (A-10) are compensated exactly by the denominators, ensuring $s_{i,i} = s_{i,j}$. From (A-10), with weights $\Omega_* = n_*(p_{*,*})^{\frac{\rho}{\rho-1}}$, the following condition must hold:

$$\frac{1}{\sum\limits_{*} \left(\Omega_* (\tau_{*,i})^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} = \frac{(\tau_{i,j})^{\frac{1}{\rho-1}}}{\sum\limits_{*} \left(\Omega_* (\tau_{*,j})^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} \tag{A-11}$$

3. In general, transport costs to different target regions are not equal, but the structure does not satisfy (A-11). Then shares $s_{i,i}$ and $s_{i,j}$ do not equalise, so it cannot be true that $p_{i,i} = p_{i,j}$.

While case 2 is mathematically feasible, the transport cost structure required to satisfy (A-11) violates the economic assumptions on transport costs. Rearranging (A-11) gives:

$$\tau_{i,j} = \left(\frac{\sum\limits_{*} \left(\Omega_{*} \left(\tau_{*,j} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}}{\sum\limits_{*} \left(\Omega_{*} \left(\tau_{*,i} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} \right)^{\rho-1}$$

Replicating (A-10) for $s_{j,j}$ and $s_{j,i}$ gives the equivalent condition (A-11) for transport cost $\tau_{j,i}$:

$$\tau_{j,i} = \left(\frac{\sum\limits_{*} \left(\Omega_{*} \left(\tau_{*,i} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}}{\sum\limits_{*} \left(\Omega_{*} \left(\tau_{*,j} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{1}{\rho}}} \right)^{\rho-1} = \frac{1}{\tau_{i,j}}$$

Under case 2, for any flow $i \rightarrow j$ subject to iceberg transport costs $\tau_{i,j} \geq 1$, the reverse flow has costs $\tau_{i,j} \leq 1$, which violates the iceberg cost assumptions that $\tau_{i,j} \geq 1 \ \forall i, j$. Only if $\tau_{i,j} = 1$ can case 2 give an acceptable outcome, but that is already covered by case 1.

Therefore, unless $\tau_{i,j} = 1 \ \forall i, j$, one cannot have $p_{i,i} = p_{i,j}$. ■

D Proof of Proposition 3 (Autarchic equilibrium)

Proposition 3: *In an autarchic setting, there always exists a firm mass n_i^a which is a solution to equations (22), (23) and (24).*

Proof:

Inverting (22) gives the following system of equations (In order to distinguish the two equations, subscripts 1 and 2 replace i, i):

$$\begin{cases} \varepsilon_1^a = -\frac{L_i}{\alpha n_i^a} \\ \varepsilon_2^a = -\sigma \left(1 - \frac{\omega_2}{s_{i,i}^a} \right) \end{cases} \tag{A-12}$$

With

$$s_{i,i}^a = \left(\frac{\eta^{1-\rho}}{n_i^a} + \left(-\omega_2 \right)^{\rho} \right)^{\frac{1}{\rho}} + \omega_2 \tag{A-13}$$

First of all, the following is true:

$$\lim_{n_i^a \rightarrow 0} \varepsilon_1^a \left(n_i^a \right) = -\infty \quad < \quad \lim_{n_i^a \rightarrow 0} \varepsilon_2^a \left(n_i^a \right) = -\sigma$$

and

$$\lim_{n_i^a \rightarrow \infty} \varepsilon_1^a \left(n_i^a \right) = 0 \quad > \quad \lim_{n_i^a \rightarrow \infty} \varepsilon_2^a \left(n_i^a \right) = -\infty$$

Functions $\varepsilon_1^a(n_i^a)$ and $\varepsilon_2^a(n_i^a)$ have the same domain, are continuous, and intersect at least once on their co-domains. Therefore, there exists at least one n_i^a which equalises $\varepsilon_1^a(n_i^a)$ and $\varepsilon_2^a(n_i^a)$ and solves the system.

Taking the first derivative of $\varepsilon_1^a(n_i^a)$ gives:

$$\frac{d\varepsilon_1^a}{dn_i,^a} = \frac{L_i}{\alpha\left(n_i^a\right)^2} \qquad > 0$$

Taking the first derivative of $\varepsilon_2^a(n_i^a)$, using the chain rule on $s_{i,i}^a$, gives:

$$\frac{d\varepsilon_2^a}{dn_{i,i}^a} = -\frac{\omega_2}{\sigma\left(s_{i,i}^a\right)^2}\frac{\eta^{1-\rho}}{\rho\left(n_{i,i}^a\right)^2}\left(\frac{\eta^{1-\rho}}{n_{i,i}^a} + (-\omega_2)^\rho\right)^{\frac{1-\rho}{\rho}} \qquad < 0$$

Both $\varepsilon_1^a(n_i^a)$ and $\varepsilon_2^a(n_i^a)$ are strictly monotonic. Therefore, n_i^a is unique. ■

E Proof of Proposition 4 (Symmetric equilibrium)

Proposition 4: *In the absence of transport costs (i.e. $\tau_{i,j} = 1\ \forall\ i,j$), there exists a symmetric equilibrium, where across regions there is: a unique price p for all varieties, a unique share variable s, a unique price index G and a unique wage w.*

Proof:

Proposition 2 establishes that under free trade, where $\tau_{i,j} = 1\ \forall\ i,j$, mill pricing $p_{i,i} = p_{i,j} = p_i$ is supported by the elasticity of demand, which implies:

$$p_{i,i} = p_{i,j} \quad \Leftrightarrow \quad \varepsilon_{i,i} = \varepsilon_{i,j} \quad \Leftrightarrow \quad s_{i,i} = s_{i,j}$$

The share variables for production flows originating in region i are given by:

$$\begin{cases} s_{i,i} = \varphi'^{-1}\left(\frac{p_{i,i}}{\tilde{P}_i}\right) \\ s_{i,j} = \varphi'^{-1}\left(\frac{p_{i,j}}{\tilde{P}_j}\right) \end{cases} \tag{A-14}$$

Given that if $p_{i,i} = p_{i,j} = p_i\ \forall\ i,j$ then $s_{i,i} = s_{i,j} = s_i\ \forall\ i,j$, it follows from (A-14) that the compositional price index in region i and j also equalise such that $\tilde{P}_i = \tilde{P}_j = \tilde{P}\ \forall\ i,j$. Equation (9) gives the manufacturing price index G in regions i and j:

$$\begin{cases} G_i = \int\limits_0^N p_{*,i}\varphi'^{-1}\left(\frac{p_{*,i}}{\tilde{P}_i}\right) dh \\ G_j = \int\limits_0^N p_{*,j}\varphi'^{-1}\left(\frac{p_{*,j}}{\tilde{P}_j}\right) dh \end{cases} \tag{A-15}$$

As for the share variables in (A-14), if $p_{i,i} = p_{i,j} = p_i \; \forall \; i,j$, then $\tilde{P}_i = \tilde{P}_j = \tilde{P} \; \forall \; i,j$ and it follows from (A-15) that $G_i = G_j = G \; \forall \; i,j$. Therefore, under mill pricing the price indexes equalise over regions:

$$p_{i,i} = p_{i,j} = p_i \quad \forall \quad i,j \quad \Leftrightarrow \quad \tilde{P}_i = \tilde{P}_j = \tilde{P} \quad \text{and} \quad G_i = G_j = G$$

Taking into account that mill pricing equalises a variety's price across regions, $p_{i,i} = p_{i,j} = p_i$, which implies $s_{i,i} = s_{i,j} = s_i$, the first order conditions for profit maximisation in regions i and j can be written as:

$$\begin{cases} p_i \left(s_i \frac{\varphi''(s_i)}{\varphi'(s_i)} + 1 \right) = \beta w_i \\ p_j \left(s_j \frac{\varphi''(s_j)}{\varphi'(s_j)} + 1 \right) = \beta w_j \end{cases} \tag{A-16}$$

One can see from (A-16) that if the mill price of a variety is the same in each region, $p_i = p_j$, then wgaes must also be equal across regions, so $w_i = w_j$.

$$p_i = p_j \quad \Leftrightarrow \quad w_i = w_j$$

It remains to show that either $p_i = p_j$ or $w_i = w_j$ occur under free trade. Taking into the mill-pricing behaviour of firms in free trade, equation (11) can be used to determine the profit made by the representative firm in any two regions i and j:

$$\begin{cases} \pi_i = p_i \sum_j m_{i,j} - w_i \left(\alpha + \beta \sum_j m_{i,j} \right) \\ \pi_j = p_j \sum_i m_{j,i} - w_j \left(\alpha + \beta \sum_i m_{j,i} \right) \end{cases} \tag{A-17}$$

The individual output flow towards a target region j, $m_{i,j}$, is given by equation (7). Combined with the equalisation of shares implied by mill pricing, this gives:

$$m_{i,j} = s_i M_j \tag{A-18}$$

These can be replacing into the profit equations (A-17). Taking into account the fact that firm entry ensures zero profits and rearranging gives:

$$\begin{cases} \sum_j M_j = \frac{\alpha w_i}{s_i(p_i - \beta w_i)} \\ \sum_i M_i = \frac{\alpha w_j}{s_j(p_j - \beta w_j)} \end{cases} \tag{A-19}$$

The left hand side of these expressions is the same, as summing the manufacturing aggregate over regions gives the same result regardless of the nature of the indexing. Equalising the right hand sides gives:

$$\frac{\alpha w_i}{s_i\left(p_i-\beta w_i\right)}=\frac{\alpha w_j}{s_j\left(p_j-\beta w_j\right)} \tag{A-20}$$

The equalisation of price indices mean that equation (A-14) can be expressed as $s_i = \varphi'^{-1}(p_i/\tilde{P})$. The share for a variety produced in i is therefore a function of the regional price p_i only, so that $s_i = s(p_i)$. Therefore, it follows from equation (A-16) that for all regions i, the wages are also a function of the regional price p_i only, so that $w_i = w(p_i)$. Inserting this in (A-20) gives the following relation:

$$\frac{\alpha w\left(p_i\right)}{s\left(p_i\right)\left(p_i-\beta w\left(p_i\right)\right)}=\frac{\alpha w\left(p_j\right)}{s\left(p_j\right)\left(p_j-\beta w\left(p_j\right)\right)} \tag{A-21}$$

For this relation to hold requires $p_i = p_j$. Therefore regional manufacturing and factor prices equalise. $\blacksquare$